

Suppressed θ_{13} In A Democratic Approach

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Abstract

Within a democratic approach based on discrete symmetries, we show how the charged lepton mass hierarchies and bilarge neutrino mixings can be realized. For the third mixing angle we find $\theta_{13} \sim \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} \sim 0.1 - 0.2$.

1 Introduction

Recent data has provided increasing evidence for atmospheric [1] and solar [2], [3] neutrino oscillations, suggesting neutrino masses such that $\Delta m_{\text{atm}}^2 \simeq 2 \cdot 10^{-3} \text{ eV}^2$, $\Delta m_{\text{sol}}^2 \simeq 7 \cdot 10^{-5} \text{ eV}^2$, and bilarge neutrino mixings: $\sin^2 2\theta_{\mu\tau} \simeq 1$, $\sin^2 2\theta_{e\mu,\tau} \approx 0.84$. It is certainly challenging to provide a self consistent theoretical explanation of these mixing angles and mass hierarchies (including the charged fermions). The introduction of flavor symmetries which distinguish the different generations seems to be one reasonable approach. An alternative idea is the democratic approach, according to which all entries in the appropriate Yukawa matrices are of the same order of magnitude. Applying this type of construction to the neutrino sector [4]-[7], one can naturally get large angles in the lepton mixing matrix. However, with neutrino democracy one also expects the third mixing angle θ_{13} to be large. On the other hand, CHOOZ experiment [8] provides the upper bound $\theta_{13} \lesssim 0.2$, and future long-baseline experiments could access θ_{13} even down to $\sim 10^{-2}$ [9], which should put severe constraints on model building.

It is clearly desirable to have an (elegant) explanation of how θ_{13} is small within neutrino democracy. The origin of the ratio $\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} = 0.017 - 0.053$ and the hierarchies between charged lepton masses also should be simultaneously understood in this approach.

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The aim of this paper is to gain an understanding of the issues mentioned above within the MSSM framework. We present a democratic scenario based on the permutation symmetry $S_2^l \times S_3^{e^c}$ acting in the lepton sector. Starting with the charged lepton sector, we show how this symmetry, especially its breaking, can be exploited to yield an explanation of the small ratios $\frac{m_\mu}{m_\tau}$, $\frac{m_e}{m_\tau}$. Next we extend our studies to the neutrino sector and demonstrate how the large atmospheric and solar neutrino oscillations are realized. Finally, we turn to θ_{13} and show that the model predicts a suppressed (but not too small) value of³ $\theta_{13} \sim \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} = 0.13 - 0.23$. These values satisfy the CHOOZ experimental bound and hold promise that the model will be tested in the near future [9].

2 Charged Lepton Sector

We start our discussion with the charged lepton sector and the symmetries which will play a crucial role in generating natural mass hierarchies and desirable lepton mixing angles. In the left-handed lepton sector, we introduce the symmetry S_2^l which exchanges l_1 and l_3 states and leaves l_2 intact:

$$S_2^l : \quad l_1 \rightarrow l_3, \quad l_3 \rightarrow l_1, \quad l_2 \rightarrow l_2. \quad (1)$$

This symmetry it turns out guarantees the smallness of θ_{13} . Indeed, in the limit when the solar neutrino mass scale is neglected, θ_{13} turns out to be zero.

For the right handed e^c states we introduce the permutation symmetry $S_3^{e^c}$ which acts as follows:

$$S_3^{e^c} : \quad e_1^c \rightarrow e_2^c, \quad e_2^c \rightarrow e_3^c, \quad e_3^c \rightarrow e_1^c. \quad (2)$$

It turns out that in the limit of unbroken $S_2^l \times S_3^{e^c}$, only a single charged lepton (tau) acquires mass. The electron and muon acquire masses from $S_2^l \times S_3^{e^c}$ breaking. Therefore, the hierarchies m_e/m_τ , m_μ/m_τ can be controlled by the symmetry breaking pattern.

To break $S_2^l \times S_3^{e^c}$ we introduce SM singlet superfields X^l and $\mathbf{X}^{e^c} = (X_1^{e^c}, X_2^{e^c}, X_3^{e^c})$ which under S_2^l and $S_3^{e^c}$ have the following transformation properties

$$S_2^l : \quad X^l \rightarrow -X^l, \quad (3)$$

$$S_3^{e^c} : \quad X_1^{e^c} \rightarrow X_2^{e^c}, \quad X_2^{e^c} \rightarrow X_3^{e^c}, \quad X_3^{e^c} \rightarrow X_1^{e^c}. \quad (4)$$

We now write down the Yukawa interactions which are responsible for charged lepton masses and respect $S_2^l \times S_3^{e^c}$ symmetry. Those couplings that do not involve the fields X^l , $X_{1,2,3}^{e^c}$ and give dominant contribution to the charged lepton mass matrix are

$$e_1^c \begin{pmatrix} l_1 & l_2 & l_3 \\ 1 & \rho & 1 \\ 1 & \rho & 1 \\ 1 & \rho & 1 \end{pmatrix} \lambda_E h_d, \quad (5)$$

³A similar relation has previously been discussed in [10]. We thank E. Akhmedov for bringing it to our attention. Employing discrete symmetries for realizing bilarge mixings and a small θ_{13} has been advocated by several authors. See, for instance, [11].

where h_d is the down type Higgs doublet superfield and ρ, λ_E are dimensionless couplings. Due to democracy ρ is of order unity, while λ_E determines the value of the MSSM parameter $\tan \beta$. The couplings involving $X_{1,2,3}^{e^c}$ and X^l are respectively

$$\begin{aligned} & \frac{\lambda_E h_d}{M} \left[X_1^{e^c} (p e_1^c + q e_2^c + r e_3^c) + X_2^{e^c} (r e_1^c + p e_2^c + q e_3^c) + X_3^{e^c} (q e_1^c + r e_2^c + p e_3^c) \right] (l_1 + l_3) + \\ & \frac{\lambda_E h_d}{M} \left[X_1^{e^c} (\rho_1 e_1^c + \rho_2 e_2^c + \rho_3 e_3^c) + X_2^{e^c} (\rho_3 e_1^c + \rho_1 e_2^c + \rho_2 e_3^c) + X_3^{e^c} (\rho_2 e_1^c + \rho_3 e_2^c + \rho_1 e_3^c) \right] l_2, \end{aligned} \quad (6)$$

$$\begin{matrix} & l_1 & l_2 & l_3 \\ e_1^c & \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \\ e_2^c & \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \\ e_3^c & \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \end{matrix} \frac{X^l}{M} \lambda_E h_d, \quad (7)$$

where M is some cut off scale and p, q, r, ρ_i are dimensionless couplings of order unity.

Next, we assume the following VEVs for the scalar components of the singlet superfields:

$$\frac{\langle X^l \rangle}{M} \equiv \epsilon_L \ll 1, \quad (8)$$

$$\frac{\langle X_1^{e^c} \rangle}{M} \equiv \epsilon_R \ll 1, \quad \langle X_2^{e^c} \rangle = \langle X_3^{e^c} \rangle = 0. \quad (9)$$

According to the breaking pattern (8), (9), the couplings in which participate $X_{2,3}^{e^c}$, are not relevant for the mass matrix. The charged lepton mass matrix can be written as

$$m_E = \begin{matrix} & e_1 & e_2 & e_3 \\ \begin{matrix} e_1^c \\ e_2^c \\ e_3^c \end{matrix} & \begin{pmatrix} 1 + p\epsilon_R + \epsilon_L & \rho + \rho_1\epsilon_R & 1 + p\epsilon_R - \epsilon_L \\ 1 + q\epsilon_R + \epsilon_L & \rho + \rho_2\epsilon_R & 1 + q\epsilon_R - \epsilon_L \\ 1 + r\epsilon_R + \epsilon_L & \rho + \rho_3\epsilon_R & 1 + r\epsilon_R - \epsilon_L \end{pmatrix} \end{matrix} \frac{\overline{m}}{\sqrt{3(2+|\rho|^2)}}, \quad (10)$$

where $\overline{m} = \lambda_E \sqrt{3(2+|\rho|^2)} \langle h_d^{(0)} \rangle$ and a suitable normalization has been chosen. For analysis, it is convenient to write (10) as

$$m_E = \frac{\overline{m}}{\sqrt{3(2+|\rho|^2)}} (Y_0 + \epsilon_R Y_R + \epsilon_L Y_L), \quad (11)$$

where

$$Y_0 = \begin{pmatrix} 1 & \rho & 1 \\ 1 & \rho & 1 \\ 1 & \rho & 1 \end{pmatrix}, \quad Y_R = \begin{pmatrix} p & \rho_1 & p \\ q & \rho_2 & q \\ r & \rho_3 & r \end{pmatrix}, \quad Y_L = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix}. \quad (12)$$

The term proportional to Y_0 in (11) provides the leading contribution in m_E and is responsible for the tau mass m_τ . Assuming that $\epsilon_L, \epsilon_R \ll 1$, for the three eigenvalues of m_E matrix, we find

$$m_\tau \simeq \overline{m}, \quad m_\mu \sim \overline{m} \epsilon_R, \quad m_e \sim \overline{m} \epsilon_R \epsilon_L. \quad (13)$$

Thus, ϵ_R and ϵ_L are responsible for the muon and electron masses⁴ such that

$$\epsilon_R \sim \frac{m_\mu}{m_\tau} \simeq 0.06, \quad \epsilon_L \sim \frac{m_e}{m_\tau \epsilon_R} \simeq 4.7 \cdot 10^{-3}. \quad (14)$$

The unitary matrix U_e which rotates the left handed charged lepton states upon diagonalization of m_E , can be found by diagonalizing the matrix $m_E^\dagger m_E$:

$$(m_E^{diag})^2 = U_e^\dagger m_E^\dagger m_E U_e. \quad (15)$$

It is easy to see that U_e is mainly determined by Y_0 , the leading term in (11):

$$U_e = P_\rho \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{|\rho|}{\sqrt{4+2|\rho|^2}} & \frac{1}{\sqrt{2+|\rho|^2}} \\ 0 & \sqrt{\frac{2}{2+|\rho|^2}} & \frac{|\rho|}{\sqrt{2+|\rho|^2}} \\ -\frac{1}{\sqrt{2}} & -\frac{|\rho|}{\sqrt{4+2|\rho|^2}} & \frac{1}{\sqrt{2+|\rho|^2}} \end{pmatrix} + \mathcal{O}(\epsilon_R) + \mathcal{O}(\epsilon_L),$$

$$\text{with} \quad P_\rho = \text{Diag}(1, \exp[-i \cdot \text{Arg}(\rho)], 1). \quad (16)$$

The corrections of order $\sim \epsilon_R, \epsilon_L$ are subleading and will not be further considered. Anticipating, the (1,1) and (3,1) entries of the leading part of the neutral lepton mixing matrix U_ν [see eq. (25)] will have the same values as corresponding entries of U_e . Because of this, there will occur cancellations which eventually lead to the suppressed θ_{13} . This effect is due to the S_2^l symmetry⁵.

To summarize, with the help of $S_2^l \times S_3^{e^c}$ symmetry, we have gained an understanding of the hierarchies between the charged lepton masses. A non trivial mixing matrix U_e (16) is also generated and will contribute to the physical lepton mixing matrix.

3 Neutrino Sector

We introduce a single right handed neutrino N which provides the dominant contribution to the neutrino mass matrix. The couplings

$$(\lambda_N l_1 + \lambda'_N l_2 + \lambda_N l_3) N h_u - \frac{1}{2} M_N N^2, \quad (17)$$

are invariant under S_2^l , and after integrating out the N state, induce an effective dimension five operator $\frac{1}{2} l m_\nu^{(0)} l$, with

$$m_\nu^{(0)} = \begin{pmatrix} 1 & t & 1 \\ t & t^2 & t \\ 1 & t & 1 \end{pmatrix} \frac{\lambda_N^2 (h_u^0)^2}{M_N}, \quad (18)$$

⁴In (13) some coefficients of order unity appear in the expressions for m_μ and m_e which will be ignored.

⁵The same symmetry plays a crucial role if non canonical kinetic terms are included in the Lagrangian. We have checked that all results are robust even in the presence of such terms.

where $t = \frac{\lambda'_N}{\lambda_N}$. The Yukawa couplings λ_N , λ'_N are expected to be of the same order of magnitude and therefore t is of order unity.

Further, we include non renormalizable operators which provide subleading contribution to the neutrino mass matrix and, as we will see, are responsible for solar neutrino oscillations. The use of non-renormalizable operators for generating neutrino masses and mixings has been discussed in the past (for instance, see [12]). The couplings respecting S_2^l symmetry are

$$\begin{array}{c} l_1 \quad l_2 \quad l_3 \\ l_1 \begin{pmatrix} 1 & \alpha & \gamma \\ \alpha & \beta & \alpha \\ \gamma & \alpha & 1 \end{pmatrix} \frac{h_u^2}{2M_1}, \quad \begin{array}{c} l_1 \quad l_2 \quad l_3 \\ l_1 \begin{pmatrix} 1 & \bar{\alpha} & 0 \\ \bar{\alpha} & 0 & -\bar{\alpha} \\ 0 & -\bar{\alpha} & -1 \end{pmatrix} \frac{h_u^2 X^l}{2M_2^2}, \end{array} \quad (19)$$

where M_1 , M_2 are some cut off scales related to lepton number violation⁶. We assume that

$$M_1 \sim \frac{M_2^2}{\langle X^l \rangle} \sim \frac{M_N}{\epsilon}, \quad \epsilon \sim 0.1. \quad (20)$$

With this, the neutrino masses will have the hierarchical structure $m_3 > m_1, m_2$ and the value of ϵ in (20), as we will see, is dictated from the observed value of $\sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}}$. Since the dominant part (18) of m_ν is responsible for $\Delta m_{\text{atm}}^2 \simeq 2,5 \cdot 10^{-3} \text{ eV}^2$, for $\lambda_N \sim t \sim 1$ we estimate $M_N/(\sin^2 \beta) \simeq 1.8 \cdot 10^{15} \text{ GeV}$. With all this and $\langle X^l \rangle = M\epsilon_L \sim M_{\text{Pl}}\epsilon_L$, taking into account (20), one has $M_1/(\sin^2 \beta) \sim M_2/(\sin \beta) \sim 10^{16} \text{ GeV}$.

From (18), (19), the mass matrix for the light neutrinos can be written as

$$m_\nu = \begin{pmatrix} 1 & t & 1 \\ t & t^2 & t \\ 1 & t & 1 \end{pmatrix} \frac{m_0}{2+|t|^2} + \begin{pmatrix} 1 & \alpha & \gamma \\ \alpha & \beta & \alpha \\ \gamma & \alpha & 1 \end{pmatrix} \frac{m_0 \epsilon}{2+|t|^2} + \begin{pmatrix} 1 & \bar{\alpha} & 0 \\ \bar{\alpha} & 0 & -\bar{\alpha} \\ 0 & -\bar{\alpha} & -1 \end{pmatrix} \frac{m_0 \delta \epsilon}{2+|t|^2}, \quad (21)$$

where

$$m_0 = \frac{\lambda_N^2 (h_u^{(0)})^2}{M_N} (2 + |t|^2), \quad \epsilon = \frac{M_N}{\lambda_N^2 M_1}, \quad \delta = \frac{M_1 \langle X^l \rangle}{M_2^2}. \quad (22)$$

Making the shift $t \rightarrow t - \alpha\epsilon$ and then rescaling $\epsilon \rightarrow (2 + |t|^2)\epsilon$, the neutrino mass matrix reads

$$m_\nu = m_\nu^{(0)} + m_\nu^{(1)},$$

$$\text{with} \quad m_\nu^{(0)} \simeq \begin{pmatrix} 1 & t & 1 \\ t & t^2 & t \\ 1 & t & 1 \end{pmatrix} \frac{m_0}{2+|t|^2}, \quad m_\nu^{(1)} = \begin{pmatrix} 1+\delta & \tilde{\alpha} & \gamma \\ \tilde{\alpha} & \tilde{\beta} & -\tilde{\alpha} \\ \gamma & -\tilde{\alpha} & 1-\delta \end{pmatrix} m_0 \epsilon, \quad (23)$$

where $\tilde{\alpha} = \bar{\alpha}\delta$, $\tilde{\beta} = \beta - 2t\alpha + \alpha^2\epsilon$.

⁶These operators may emerge from the decoupling of heavy states with masses $M_{1,2}$.

The leading part $m_\nu^{(0)}$ is diagonalized by the transformation

$$(U_\nu^{(0)})^T m_\nu^{(0)} U_\nu^{(0)} = \text{Diag}(0, 0, m_0) , \quad (24)$$

where

$$U_\nu^{(0)} = P_t \begin{pmatrix} \frac{1}{\sqrt{2}} , & -\frac{|t|}{\sqrt{4+2|t|^2}} , & \frac{1}{\sqrt{2+|t|^2}} \\ 0 , & \sqrt{\frac{2}{2+|t|^2}} , & \frac{|t|}{\sqrt{2+|t|^2}} \\ -\frac{1}{\sqrt{2}} , & -\frac{|t|}{\sqrt{4+2|t|^2}} , & \frac{1}{\sqrt{2+|t|^2}} \end{pmatrix} ,$$

$$\text{with } P_t = \text{Diag}(1, \exp[-i \cdot \text{Arg}(t)], 1) . \quad (25)$$

Note that only with this transformation, the lepton mixing matrix $V^l = U_e^\dagger U_\nu^{(0)}$ would yield $\theta_{12} = \theta_{13} = 0$. However, the subleading term $m_\nu^{(1)}$ in (23) can provide naturally large 1-2 mixing, with θ_{13} still remaining small. To see this, we will perform the transformation $(U_\nu^{(0)})^T m_\nu U_\nu^{(0)}$, under which with redefinitions $\tilde{\alpha} \rightarrow \exp[i \cdot \text{Arg}(t)]\tilde{\alpha}$, $\tilde{\beta} \rightarrow \exp[i \cdot 2\text{Arg}(t)]\tilde{\beta}$ the neutrino mass matrix (23) becomes

$$m'_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} + \begin{pmatrix} 1 - \gamma & \frac{2\tilde{\alpha} - \delta|t|}{\sqrt{2+|t|^2}} & \frac{\sqrt{2}(\tilde{\alpha}|t| + \delta)}{\sqrt{2+|t|^2}} \\ \frac{2\tilde{\alpha} - \delta|t|}{\sqrt{2+|t|^2}} & \frac{2\tilde{\beta} + |t|^2(1+\gamma)}{2+|t|^2} & \frac{\sqrt{2}|t|}{2+|t|^2}(\tilde{\beta} - 1 - \gamma) \\ \frac{\sqrt{2}(\tilde{\alpha}|t| + \delta)}{\sqrt{2+|t|^2}} & \frac{\sqrt{2}|t|}{2+|t|^2}(\tilde{\beta} - 1 - \gamma) & \frac{\tilde{\beta}|t|^2 + 2(1+\gamma)}{2+|t|^2} \end{pmatrix} m_0 \epsilon . \quad (26)$$

From (26) one can see that the additional rotations that are needed to diagonalize the neutrino mass matrix can yield a small θ_{13} ($\tan \theta_{13} \sim \epsilon$). Also, the correction to the 2-3 rotation is suppressed by ϵ . However, the 1-2 rotation can be naturally large. The matrix m'_ν is diagonalized by

$$(U_\nu^{(1)})^T m'_\nu U_\nu^{(1)} = \text{Diag}(m_1, m_2, m_0) \quad (27)$$

with

$$U_\nu^{(1)} \simeq \begin{pmatrix} c_\theta & s_\theta & |\kappa|\epsilon \\ -s_\theta e^{i\phi} & c_\theta e^{i\phi} & |\kappa'|\epsilon e^{i(\phi_\kappa - \phi_{\kappa'})} \\ -(\kappa c_\theta - \kappa' s_\theta e^{i\phi})\epsilon & -(\kappa s_\theta + \kappa' c_\theta e^{i\phi})\epsilon & e^{i\phi_{\kappa'}} \end{pmatrix} , \quad (28)$$

where $c_\theta \equiv \cos \theta$, $s_\theta \equiv \sin \theta$ and

$$\tan 2\theta = -\frac{2|b|}{|a - ce^{i2\phi}|} , \quad \kappa = \frac{\sqrt{2}(\tilde{\alpha}|t| + \delta)}{\sqrt{2+|t|^2}} , \quad \kappa' = \frac{\sqrt{2}|t|}{2+|t|^2}(\tilde{\beta} - 1 - \gamma) , \quad (29)$$

with

$$a = 1 - \gamma , \quad b = \frac{2\tilde{\alpha} - \delta|t|}{\sqrt{2+|t|^2}} , \quad c = \frac{2\tilde{\beta} + |t|^2(1+\gamma)}{2+|t|^2} , \quad \phi_{\kappa, \kappa'} = \text{Arg}(\kappa, \kappa') . \quad (30)$$

The phase ϕ is determined by

$$\sin(\phi - \phi_b + \phi_c) = \left| \frac{a}{c} \right| \sin(\phi + \phi_b + \phi_a) , \quad \text{where} \quad \phi_{a,b,c} = \text{Arg}(a, b, c) . \quad (31)$$

The masses of the three neutrino eigenstates are⁷

$$\begin{aligned} m_1 &\simeq \left[ac_\theta^2 - 2bs_\theta c_\theta e^{i\phi} + cs_\theta^2 e^{i2\phi} \right] m_0 \epsilon , \\ m_2 &\simeq \left[as_\theta^2 + 2bs_\theta c_\theta e^{i\phi} + cc_\theta^2 e^{i2\phi} \right] m_0 \epsilon , \\ m_3 &\simeq m_0 . \end{aligned} \quad (32)$$

From (32),

$$\begin{aligned} \Delta m_{\text{atm}}^2 &= |m_3|^2 - |m_2|^2 \simeq |m_0|^2 , \\ \Delta m_{\text{sol}}^2 &= |m_2|^2 - |m_1|^2 \sim |m_0|^2 \epsilon^2 . \end{aligned} \quad (33)$$

Therefore, the estimated value of ϵ is

$$\epsilon \sim \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} = 0.13 - 0.23 . \quad (34)$$

Now let us discuss the lepton mixing angles. Upon its full diagonalization the neutrino mass matrix was transformed as

$$(m_\nu)^{\text{diag}} = U_\nu^T m_\nu U_\nu , \quad (35)$$

where

$$U_\nu = U_\nu^{(0)} U_\nu^{(1)} . \quad (36)$$

The lepton mixing matrix is given by

$$(V^l)_{\alpha i} = (U_e^\dagger U_\nu)_{\alpha i} , \quad (37)$$

where α refers to the flavor index ($\alpha = e, \mu, \tau$) and $i = 1, 2, 3$ denotes the light neutrino mass eigenstates. One finds

$$V_{\alpha i}^l \simeq \begin{pmatrix} c_\theta , & s_\theta , & \frac{|\kappa| \epsilon}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} e^{i\phi_\kappa} \\ -\frac{2e^{i\omega} + |t\rho|}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} s_\theta e^{i\phi} , & \frac{2e^{i\omega} + |t\rho|}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} c_\theta e^{i\phi} , & \frac{\sqrt{2}(|t|e^{i\omega} - |\rho|)}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} e^{i\phi_\kappa} \\ \frac{\sqrt{2}(|t| - |\rho|e^{i\omega})}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} s_\theta e^{i\phi} , & -\frac{\sqrt{2}(|t| - |\rho|e^{i\omega})}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} c_\theta e^{i\phi} , & \frac{2 + |t\rho|e^{i\omega}}{\sqrt{(2+|t|^2)(2+|\rho|^2)}} e^{i\phi_\kappa} \end{pmatrix}_{\alpha i} , \quad (38)$$

where $\omega = \text{Arg}(\rho) - \text{Arg}(t)$. From (38), we find

$$\tan \theta_{\mu\tau} \simeq \sqrt{2} \left| \frac{t - \rho}{2 + t\rho^*} \right| , \quad \tan \theta_{e\mu,\tau} \simeq \tan \theta . \quad (39)$$

⁷In general, $m_{1,2,3}$ are complex and an additional diagonal phase matrix is needed to make them real. However, this phase matrix does not play a role in neutrino oscillations and can be ignored.

According to the democratic approach, we have $t, \rho, \tan \theta \sim 1$, so that

$$\sin^2 2\theta_{\mu\tau} \sim 1, \quad \sin^2 2\theta_{e\mu,\tau} \sim 1. \quad (40)$$

Thus, bilarge neutrino mixing is realized. For the remaining angle θ_{13} , from (38) taking (34) into account,

$$\theta_{13} \equiv V_{e3}^l \simeq |\kappa|\epsilon \sim |\kappa| \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}}. \quad (41)$$

Thus, θ_{13} is naturally suppressed ($\sim \epsilon$), but seems within reach of the next round experiments.

4 Conclusions

By exploiting a democratic approach supplemented by suitable symmetries, we have attempted to obtain an understanding of the charged lepton mass hierarchies, bilarge neutrino mixing as well as a suppressed mixing angle θ_{13} . Since we studied this in the MSSM framework augmented with singlet states, the lepton sector does not have any impact on quark masses and their mixings. Because of this, the quark sector can also nicely blend with neutrino democracy. However, this will change if some GUT scenario such as $SU(5)$ and $SO(10)$ is considered, where the quark and lepton mass matrices can be related to each other. For realistic pattern of fermion masses and mixings some extensions will become necessary [5], [6].

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